Bounds on Membership Uncertainty: Exercises

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A Stirling Bound The power series expansion of the exponential function is

$$\exp(x) \;=\; 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^m}{m!} + \cdots$$

(If you didn't happen to know this, you can see that it must be the case by expanding the Taylor polynomial of $\exp(x)$ in $x = 0$, or by thinking about what kind of polynomial would satisfy the differential equation $p'(x) = p(x)$.)

Use this expansion to prove that

1. $k! \;\leq\; (k/e)^k$
2. $k! \;\leq\; 2\,(k/e)^k$

Big Bonferroni Correction A sample of size $2t = 2\times 10^4$ is drawn from some distribution, and this sample is then randomly split up into two half-samples of size $t = 10^4$.

1. For any specific event A, these two half-samples define two frequencies, $f_1(A)$ and $f_2(A)$. Find an explicit upper bound on the probability that $|f_1(A) - f_2(A)| > 0.1$.
2. We now make such a comparison for each of $\Phi(3, 2\times 10^4)$ different sets. Find an explicit upper bound on the probability that $|f_1(A) - f_2(A)| > \varepsilon$ for at least one A.