ILLC Project Course in Statistical Learning Theory

Mathias Winther Madsen

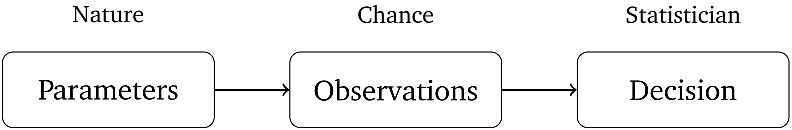
mathias.winther@gmail.com

Institute for Logic, Language, and Computation
University of Amsterdam

January 2015

Worst-Case Analysis

A decision problem:



+ a loss function.

Worst-Case Analysis

The Bernoulli game:

1. Nature chooses a coin bias θ and a precision ε.
2. The statistician chooses a sample size t.
3. Chance flips Nature's coin t times.

The the frequency $\hat{\theta}$ of successes in the sample is computed, and statistician is paid 1 cent if $|\theta - \hat{\theta}| \leq \varepsilon$.

Worst-Case Analysis

The no-free-lunch game

1. Nature chooses a coin bias θ.
2. Chance flips the coin $2t$ times and reveals the first t results,

$$x_1, x_2, \ldots, x_t,$$

   to the statistician.
3. The statistician makes a guess at the last t outcomes,

$$\hat{x}_{t+1}, \hat{x}_{t+2}, \ldots, \hat{x}_{2t}.$$

These guesses are compared to reality, and the statistician loses 1 cent for each wrong guess and wins 1 cent for each correct.

Worst-Case Analysis

The no-free-lunch game

1. Nature chooses a binary string

$$x_1, x_2, \ldots, x_t, x_{t+1}, x_{t+2}, \ldots, x_{2t},$$

   and the first t elements are revealed to the statistician.
2. The statistician tries to guess at the last t outcomes,

$$\hat{x}_{t+1}, \hat{x}_{t+2}, \ldots, \hat{x}_{2t}.$$

These guesses are compared to reality, and the statistician loses 1 cent for each wrong guess and wins 1 cent for each correct.

Worst-Case Analysis

The Vapnik-Chervonenkis game:

1. Nature chooses a probability distribution P over the sample space Ω and a precision level ε.
2. The statistician chooses a sample size $2t$.
3. Chance draws $2t$ samples from Ω according to P.

We compute the maximum difference in frequency,

$$\nu \;=\; \sup_{A} \left| f_1(A) - f_2(A) \right| ,$$

and the statistician wins 1 cent if $\nu \leq \varepsilon$.

The Uniform Law of Large Numbers

Theorem

A portfolio of prediction methods will have the same error rate on a training set and a test if the portfolio is "small."