ILLC Project Course in Statistical Learning Theory

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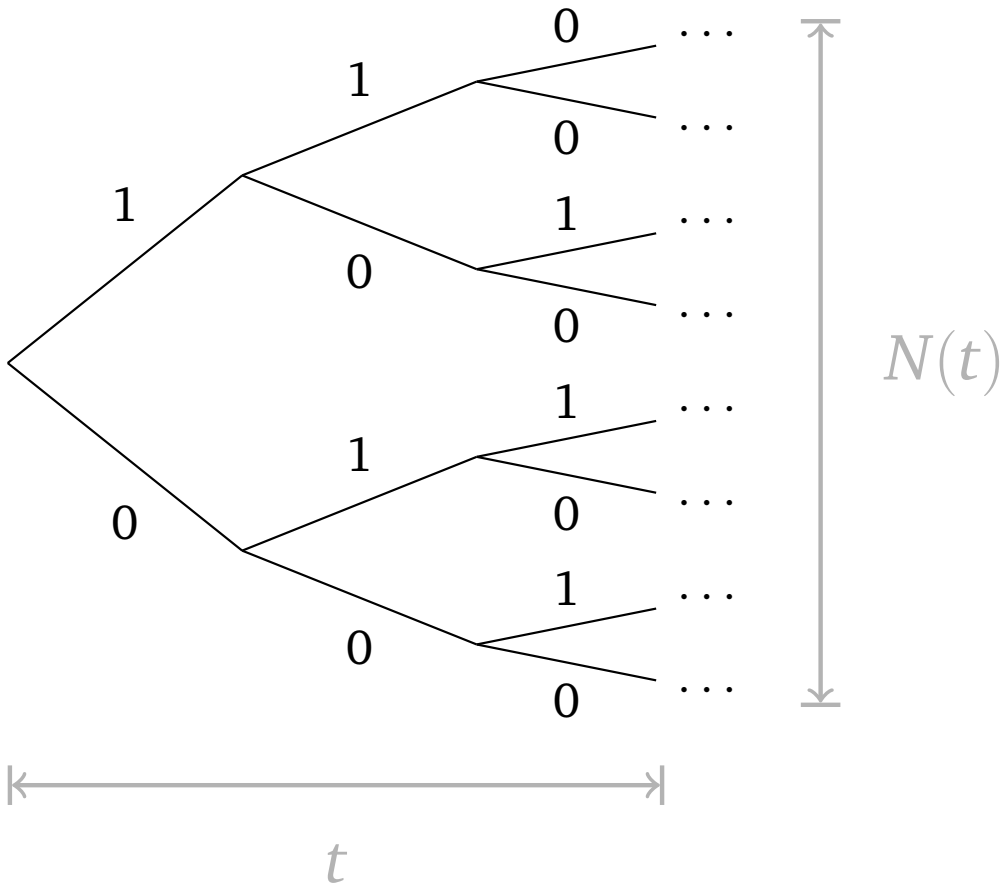
Nondeterministic Processes

Definition

A nondeterministic process is a set of infinite sequences from a finite alphabet $\mathbb{A}$ (typically the binary alphabet $\mathbb{A} = \{0, 1\}$).



Nondeterministic Processes

The fencepost process

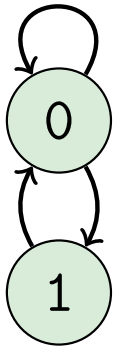
All sequences of equally spaced `1`s, e.g.,

`0001000100010001000100 . . .`

The Morse code process

All sequences without consecutive `1`s, e.g.,

`0010100100001010101000 . . .`



The outspender process

All sequences of increasingly long runs of `0`s and `1`s, e.g.,

`0011110000000111111111 . . .`

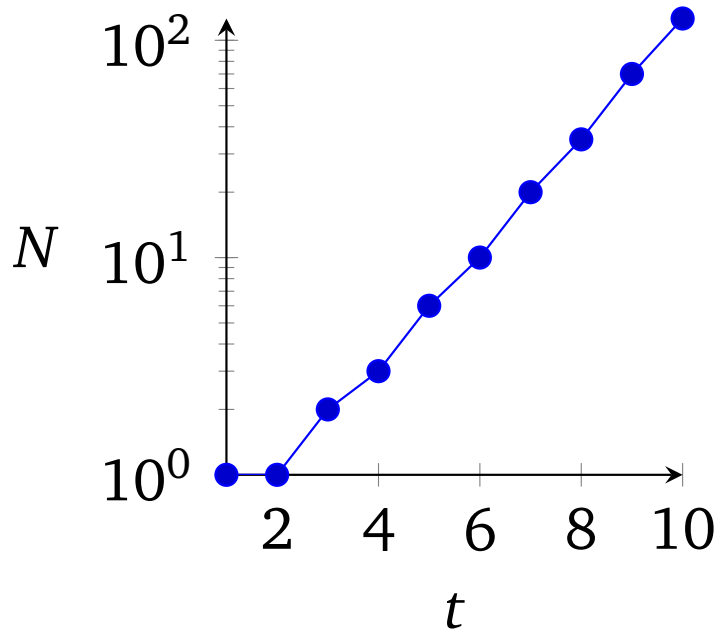
Entropy Rates

Definition

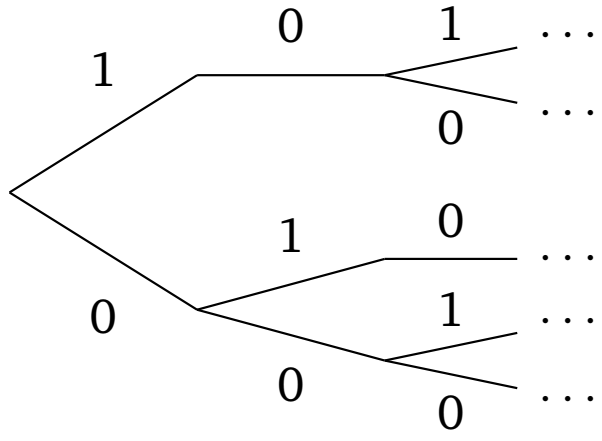
The **entropy rate** of a non-deterministic process is

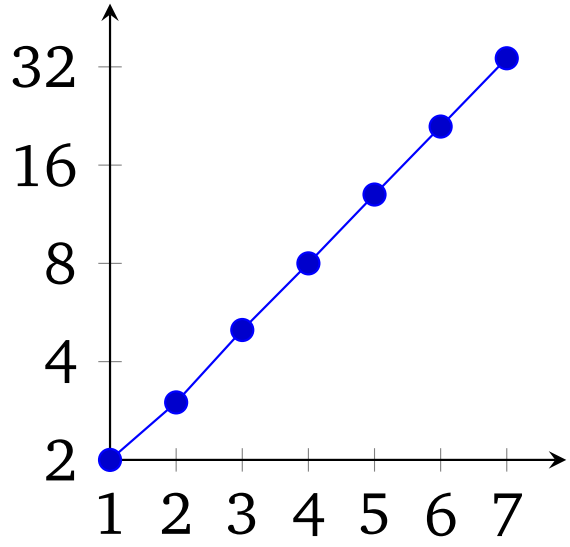
$$H = \lim_{t \to \infty} \frac{\log N(t)}{t}$$

whenever this limit is well-defined.



Entropy Rates





t	1	2	3	4	5	$\cdots$	100	$\cdots$
$N(t)$	2	3	5	8	13	$\cdots$	$9 \cdot 10^{20}$	$\cdots$
$\log N(t)$	1.0	1.6	2.3	3.0	3.7	$\cdots$	69.7	$\cdots$
$\log N(t)/t$	1.0	0.8	0.8	0.8	0.7	$\cdots$	0.7	$\cdots$

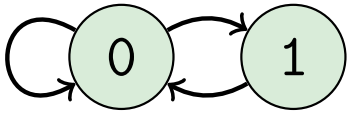
Entropy Rates

The fencepost process

The fencepost process grows as $N(t) = t + 1$ and thus has an entropy rate of $H = 0$.

The Morse code process

The More code process has an entropy rate of $H \approx .694$.



The outspender process

The outspender process has an entropy rate of $H = 0$.