The Law of Large Numbers: Solutions

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A Markovian Variable Such a variable must have the cumulative distribution

$$\Pr\{X \leq b\} \;=\; 1 - \frac{E[X]}{b} \qquad (b \geq 0).$$

By differentiating, we can find that it must have the probability density

$$p(b) \;=\; \frac{E[X]}{b^2} \qquad (b \geq 0),$$

which happens to be a Pareto distribution.

Coin-Flipping Chebychev When X is a Bernoulli variable with mean μ,

$$Var[X] \;=\; \mu(1-\mu) \;\leq\; \frac{1}{4}.$$

Plugging this bound on the variance into Chebyshev's inequality gives the desired result.

Coin-Flipping Statistics

1. $$t \;=\; \frac{1}{4\varepsilon^2\alpha} \;=\; 5 \times 10^4.$$

2. $$\varepsilon \;=\; \frac{1}{2\sqrt{t\alpha}} \;\approx\; 7.1 \times 10^{-2}.$$

3. $$\alpha \;=\; \frac{1}{4\varepsilon^2 t} \;=\; 2.5.$$