The Law of Large Numbers: Exercises

Mathias Winther Madsen

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A Markovian Variable Exhibit a variable that meets the Markov bound

$$\Pr\{X > b\} \;\leq\; \frac{E[X]}{b}$$

exactly for all $b \geq 0$ and sketch its density function.

Coin-Flipping Chebyshev Suppose that S_t is a sum of t independent and identically distributed Bernoulli variables (i.e., coin flipping variables with values 0 and 1). Suppose further that their shared mean is $E[X] = \mu$.

Prove that

$$\Pr\left\{\left|\frac{S_t}{t} - \mu\right| > \varepsilon\right\} \;\leq\; \frac{1}{4t\varepsilon^2}.$$

Coin-Flipping Inference We flip a coin t times in order to estimate how biased it is, aiming for a precision level of $\varepsilon > 0$ and an error probability of $\alpha > 0$. Use the previous result to answer the following questions:

1. How many times should we flip the coin in order to achieve a precision of $\varepsilon = 10^{-2}$ and error probability of $\alpha = 1/20$?
2. If we flip the coin $t = 10^3$ times and want to the error probability to be less than $\alpha = 1/20$, what precision level can we can obtain?
3. If we flip the coin $t = 10^3$ times, what is the probability that the empirical frequency of heads deviates from the probability by more than $\varepsilon = 10^{-2}$?