

# VC Dimension: Exercises

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**Trivial Portfolio** Give an example of a portfolio with a VC dimension of 0.

**Windows** Find the VC dimension of the portfolio whose elements are unions of two rectangles in the plane:



**Linearly Ordered Portfolios (Anthony and Biggs, Exs. 7.8 and 7.9)**

A portfolio  $S$  is linearly ordered if any pair of sets  $A_1, A_2 \in S$  satisfy either

$$A_1 \subseteq A_2 \quad \text{or} \quad A_1 \supseteq A_2.$$

1. Prove that if a portfolio is linearly ordered, then its VC dimension is 1.
2. Prove that if a portfolio has a VC dimension of 1, and if it contains  $\Omega$  and  $\emptyset$ , then it is linearly ordered.

Both of these results are contained in Wenocur and Dudley: “Some Special Vapnik-Chervonenkis Classes” (*Discrete Mathematics*, 1981).