Worst-Case Analysis: Exercises

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Minimax Cookies Nature runs a factory that produces cookies, each of which are poisoned with probability p. The factory can set this parameter as they like, and you don't know the current value.

You buy two cookies from this factory and send a randomly selected one to the lab to have it tested for poison. After obtaining the result, you can either eat the other cookie or throw it away.

We will assume that you lose $L = 1$ point when you eat a poisoned cookie, $L = 0$ when you don't eat anything, and $L = -3$ points when you eat one that isn't poisoned. (You really like cookies.)

1. How many different strategies can you adopt in this decision problem?
2. For each of these strategies, draw a graph showing the corresponding risk (average loss) as a function of p.
3. For each strategy, determine what the worst-case scenario is, and identify the strategy that leads to the lowest worst-case risk.

Dunn-Šidák correction A series of t independent experiments are performed, each of which fail with probability α.

1. What is the probability that at least one of these experiment fails?
2. At what level should you set α if you want this probability to be 0.05?

Bonferroni Correction In order to simultaneously estimate the probabilities of three different sets, you take t samples from a sample space Ω. Taken individually, each one of the three estimates will be wrong with probability α, but these failures may be dependent because of the way the sets overlap.

1. Find an upper bound on the probability that at least one of your estimates is wrong.
2. When is this bound most and least accurate?
3. Using this bound, find a value of α for which the misestimation probability is less than 0.05.